

A model for Dirac neutrino mass matrix with only four parameters

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Oklahoma State University, Stillwater, OK 74078-3072, USA.**Abstract**

The exchange symmetry between the muon neutrino and the tau neutrino for the neutrino mass matrix has been very useful in understanding the near maximal atmospheric neutrino mixing angle. However, this symmetry can not be imposed at the Lagrangian level, since the charged lepton partners, muon and tau do not satisfy this symmetry. We extend the Standard model to include three right handed singlet neutrinos, and impose the most general symmetry between $\nu_{\mu R}$ and $\nu_{\tau R}$ sectors followed by a CP transformation of the leptonic sector at the Lagrangian level. This symmetry does not affect the charged leptons. With the additional assumption of the hermiticity of the ensuing Dirac neutrino mass matrix, we get a 4 parameter neutrino mass matrix in good agreement with the available neutrino data for the inverted neutrino mass hierarchy. The model also predicts the values of the three neutrino masses, and the leptonic CP violating phase which can be tested in the upcoming neutrino experiments.

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The mixing between different neutrino flavors was first hinted by the deficit of solar neutrino flux as measured in Earth. The solar neutrino deficit can be explained if we assume non-zero neutrino masses, mixings and hence, oscillation between different neutrino flavors. During last two decades different experiments on atmospheric (ν_μ and $\bar{\nu}_\mu$) neutrinos (Super-K [1], K2K [2], MINOS [3]), solar (ν_e) neutrinos (SNO [4], Super-K [5], KamLAND [6]) as well as reactor/accelerator ($\bar{\nu}_e/\nu_\mu$) neutrinos (Daya Bay [7], RENO [8], Double Chooz [9], T2K [10], NO ν a [11]) provided us convincing evidences for non-zero neutrino masses and mixings. All currently available data on the oscillations can be described assuming 3-flavor (ν_e , ν_μ and ν_τ) neutrino mixing in vacuum. In the basis where the weak interaction is flavor diagonal and universal, the mass eigenstates (ν_1 , ν_2 and ν_3) are related to the weak (flavors) eigenstates (ν_e , ν_μ and ν_τ) as follows,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad (1)$$

where, U is the 3×3 neutrino mixing matrix [12, 13]. For Dirac neutrinos the mixing matrix U can be parametrized by 3 angles (θ_{12} , θ_{13} and θ_{23}) and one CP violating phase (δ):

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (2)$$

where, $c_{ij} = \text{Cos}\theta_{ij}$ and $s_{ij} = \text{Sin}\theta_{ij}$. A global analysis of neutrino oscillations data from different experiments give the best fit values for the three mixing angles and two squared-mass differences [14], $\Delta m_{ij}^2 = m_i^2 - m_j^2$. However there are several important parameters yet to be measured. These include the value of the CP phase(s) which will determine the magnitude of CP violation in the leptonic sector and the sign of Δm_{32}^2 which will determine whether the neutrino mass hierarchy is normal or inverted. Moreover, we also don't know yet if the neutrinos are Majorana or Dirac particles.

The best fit values of the three mixing angles and two squared-mass differences along with their 3σ allowed range are presented in Table 1. The experimental data in Table 1 shows two important properties: (i) There is a $\mathcal{O}(10^2)$ hierarchy in the squared-mass differences and (ii) The atmospheric and solar mixing angles (θ_{12} and θ_{23}) are large whereas the reactor mixing angle (θ_{13}) is very small. It is well known that the presence of tiny quantities or hierarchies indicates towards a protection symmetry in underlying scenario [15]. Example of one such well studied symmetry, in the context of neutrino physics, is the invariance of flavor neutrino mass matrix under interchange of ν_μ and ν_τ [16, 17, 18, 19, 20, 21]. It is easy to see from Eq. 2 that the exact μ - τ symmetry of the neutrino mixing matrix demands $s_{23}^2 = 0.5$ and $s_{13} = 0$. Table 1 shows that $s_{23}^2 = 0.5$ is still within the 3σ of the central value however, $s_{13} = 0$ is already ruled out with more than 5σ C.L. Moreover, the charged leptons and left handed neutrinos are in the $SU(2)_L$ doublets and thus, the μ - τ symmetry respected by the neutrinos should be respected by the charged leptons. However, the charged leptons clearly violate these symmetries at the Lagrangian level. Therefore, one can only impose μ - τ symmetry as a symmetry of neutrino mass matrix not as a symmetry of the Lagrangian. This fact apparently disfavors the requirement of the μ - τ symmetry.

In this work, we have enlarged the SM field content by introducing three right handed $SU(2)_L$ singlet neutrino fields (ν_{eR} , $\nu_{\mu R}$ and $\nu_{\tau R}$). We have also considered Yukawa terms

Parameter	best-fit ($\pm\sigma$)	3σ
$\Delta m_{21}^2 [10^{-5} eV^2]$	$7.53^{+0.26}_{-0.22}$	6.99 - 8.18
$\Delta m^2 [10^{-3} eV^2]$	$2.43^{+0.06}_{-0.10} (2.42^{+0.07}_{-0.11})$	2.19(2.17) - 2.62(2.61)
$\sin^2 \theta_{12}$	$0.307^{+0.018}_{-0.016}$	0.259 - 0.359
$\sin^2 \theta_{23}$	$0.386^{+0.024}_{-0.021} (0.392^{+0.039}_{-0.022})$	0.331(0.335) - 0.637(0.663)
$\sin^2 \theta_{13}$	$0.0241 \pm 0.0025 (0.0244^{+0.0023}_{-0.0025})$	0.0169(0.0171) - 0.0313(0.0315)

Table 1: The best-fit values and 3σ allowed ranges of the 3-neutrino oscillation parameters. The values (values in brackets) correspond to normal neutrino mass hierarchy (NH) i.e., $m_1 < m_2 < m_3$ (inverted neutrino mass hierarchy (IH) i.e., $m_3 < m_1 < m_2$). The definition of Δm^2 used is $\Delta m^2 = m_3^2 - (m_2^2 + m_1^2)/2$. Thus $\Delta m^2 = \Delta m_{31}^2 - m_{21}^2/2$ if $m_1 < m_2 < m_3$ and $\Delta m^2 = \Delta m_{32}^2 + m_{21}^2/2$ for $m_3 < m_1 < m_2$.

for the neutrinos in order to give them Dirac masses⁴. In this frame work, we can demand a invariance of flavor neutrino mass terms under the interchange of the right handed muon neutrino ($\nu_{\mu R}$) and tau neutrino ($\nu_{\tau R}$). The RH charged leptons and neutrinos are singlet under $SU(2)_L$ and thus they do not form a multiplet. Therefore, we can invoke any symmetry in the RH neutrino sector without imposing that symmetry in the charged lepton sector. If any symmetry exists in the Dirac neutrino mass matrix under interchange of $\nu_{\mu R}$ - $\nu_{\tau R}$ then this will be symmetry of the whole Lagrangian. We have constructed the different Dirac neutrino mass matrices assuming different kinds of symmetries in the $\nu_{\mu R}$ and $\nu_{\tau R}$ sector and tried to fit the experimentally observed quantities. Finally, we end up with a four parameter Dirac neutrino mass matrix which is based on the assumption of the Hermiticity⁵ of the Dirac neutrino mass matrix and a particular symmetry between $\nu_{\mu R}$ and $\nu_{\tau R}$. We have also shown that assuming IH in the neutrino sector, this four parameter neutrino mass matrix is consistent with the observed values of the three mixing angles and two squared-mass differences listed in Table 1, and also makes definite predictions for the values of the three neutrino masses and the leptonic CP violating phase .

The most general Dirac neutrino mass matrix contain 9 complex parameters and can be written as:

$$\mathbf{M}_\nu = \begin{pmatrix} m_{eLeR} & m_{eL\mu R} & m_{eL\tau R} \\ m_{\mu LeR} & m_{\mu L\mu R} & m_{\mu L\tau R} \\ m_{\tau LeR} & m_{\tau L\mu R} & m_{\tau L\tau R} \end{pmatrix}. \quad (3)$$

⁴If the neutrinos get mass via the Yukawa couplings with the SH Higgs then the order of the neutrino Yukawa coupling should be about 10^{-12} . However, there are interesting studies in the literature [22] which assume a discrete Z_2 symmetry and a second Higgs doublet with vacuum expectation value in the eV to keV range, in order to generate sub eV scale Dirac type neutrino masses with a Yukawa coupling of the order of charged lepton Yukawa coupling.

⁵It is important to note that the assumption of Hermiticity is somewhat ad hoc i.e., Hermiticity of neutrino mass matrix is not an outcome of symmetry argument. However, we have shown in the following that with this assumption, the existing neutrino data can completely determine the mass matrix for the Dirac neutrinos with particular predictions for the neutrino masses and the CP violating phase which can be tested at the ongoing and future neutrino experiments. Therefore, in our analysis, the assumption of hermiticity of neutrino mass matrix is a purely phenomenological assumption. However, in the future, there might be some compelling theoretical framework which requires the hermiticity of neutrino mass matrix.

On this 18 parameter Dirac neutrino mass matrix, we have imposed the following conditions:

- We have assumed the hermiticity of the neutrino mass matrix. As a result of this assumption, the diagonal elements of Eq. 3 become real and off-diagonal elements become complex conjugate of each other: $m_{\mu_L e_R} = m_{e_L \mu_R}^*$, $m_{\tau_L e_R} = m_{e_L \tau_R}^*$ and $m_{\tau_L \mu_R} = m_{\mu_L \tau_R}^*$. Therefore, after demanding the hermiticity, we have a 9 parameter neutrino mass matrix.

The hermitian neutrino mass matrix is given in the flavor basis by

$$M_\nu = U_\nu \mathbf{M}_\nu^{diag} U_\nu^\dagger, \quad (4)$$

where, M_ν^{diag} is the diagonal neutrino mass matrix in the mass basis. Two squared-mass differences of the neutrinos are known from the experiments. Therefore, M_ν^{diag} can be constructed with only one mass as unknown. For IH, the diagonal neutrino mass matrix is given by,

$$\mathbf{M}_\nu^{diag} = \begin{pmatrix} \sqrt{m_3^2 + 0.002315} & 0 & 0 \\ 0 & \sqrt{m_3^2 + 0.00239} & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad (5)$$

where, m_3 is the unknown mass and we have used the central values of the squared-mass differences listed in Table 1 for IH. In the mixing matrix U , there are three angles and one phase. The mixing angles are already measured (see Table 1 for their central values and 3σ range) with good precision. In our analysis, we have considered the IH central values for the s_{12}^2 and s_{13}^2 . However, we have considered $s_{23}^2 = 0.5$ which is not the central value but well within 3σ of the central value.

If we assume one particular neutrino mass hierarchy, there are still two quantities unknown in for the Dirac neutrinos namely, the mass m_3 in the diagonal mass matrix and the CP violating phase (δ) in the mixing matrix. In our analysis, we have scanned unknown parameters (m_3 and δ) over a range of values and tried to find out a constrained phenomenological neutrino mass matrix which is consistent with the 5 experimental results (three mixing angles and two squared-mass differences). Our phenomenological results are summarized in the following:

- In Fig. 1, we have presented $m_{\mu_L \mu_R}$ and real part of $-m_{\mu_L \tau_R}$ elements of the Dirac neutrino mass matrix in Eq. 3 as a function of m_3 . The other free parameter δ was randomly varied between 0 and π . Fig. 1 shows that two curves intersects each other at $m_3 = -1.198 \times 10^{-3} \text{ eV}$.
- In Fig. 2, we have presented real and imaginary parts of the elements $m_{e_L \mu_R}$ and $m_{e_L \tau_R}$ (left panel) and diagonal elements $m_{\mu_L \mu_R}$ and $m_{\tau_L \tau_R}$ (right panel) of the Dirac neutrino mass matrix in Eq. 3 as a function of δ for $m_3 = -1.198 \times 10^{-3} \text{ eV}$. Fig. 2 shows that a constrained neutrino mass matrix is obtained for $\delta = \pi/2$ and $m_3 = -1.198 \times 10^{-3} \text{ eV}$. The numerical form of the mass matrix in the flavour basis for $\delta = \pi/2$ and $m_3 = -1.198 \times 10^{-3} \text{ eV}$ is given by,

$$\begin{pmatrix} 4.72 \times 10^{-2} & 2.49 \times 10^{-4} - 5.37 \times 10^{-3}i & -2.49 \times 10^{-4} - 5.37 \times 10^{-3}i \\ 2.49 \times 10^{-4} + 5.37 \times 10^{-3}i & 2.43 \times 10^{-2} & -2.43 \times 10^{-2} \\ -2.49 \times 10^{-4} + 5.37 \times 10^{-3}i & -2.43 \times 10^{-2} & 2.43 \times 10^{-2} \end{pmatrix}, \quad (6)$$

It is important to note that the mass matrix in Eq. 6 is a four parameter matrix can be written

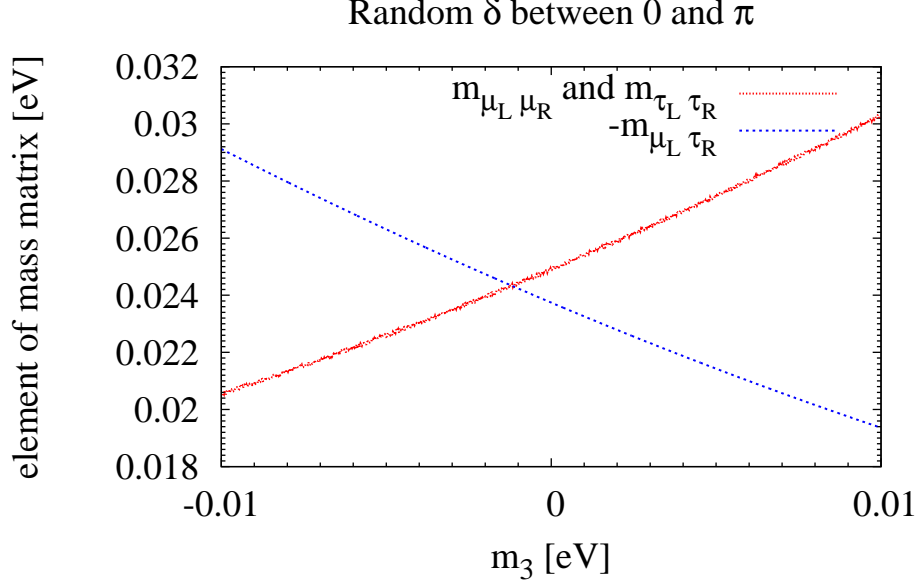


Figure 1: The elements $m_{\mu_L \mu_R}$ and real part of $-m_{\mu_L \tau_R}$ of the Dirac neutrino mass matrix in Eq. 3 as a function of m_3 . The other free parameter δ was randomly varied between 0 and π . We have used IH central values for the Δm_{21}^2 , Δm^2 , s_{12}^2 and s_{13}^2 from Table 1 and for s_{23}^2 , we choose $s_{23}^2 = 0.5$.

as,

$$\mathbf{M}_\nu^{pheno} = \begin{pmatrix} a & be^{i\eta} & -be^{-i\eta} \\ be^{-i\eta} & c & -c \\ -be^{i\eta} & -c & c \end{pmatrix}, \quad (7)$$

with $a = 4.72 \times 10^{-2}$, $b = 5.38 \times 10^{-3}$, $c = 2.43 \times 10^{-2}$ and $\eta = 272.6^\circ$. We now search for symmetry in the $\nu_{\mu R} - \nu_{\tau R}$ sector which is consistent with the structure of the phenomenological neutrino mass matrix in Eq. 7.

The most general transformation in the $\nu_{\mu R} - \nu_{\tau R}$ sector can be written as,

$$\Psi_R = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \rightarrow \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & pe^{i\phi_2} & -qe^{-i\phi_3} \\ 0 & qe^{i\phi_3} & pe^{-i\phi_2} \end{pmatrix} \Psi_R \rightarrow U_R \Psi_R, \quad (8)$$

where, $p^2 + q^2 = 1$ and ϕ_1 , ϕ_2 and ϕ_3 are the arbitrary phases. As already discussed in the beginning of this paper, we do not want to introduce any symmetry in $\nu_{\mu L} - \nu_{\tau L}$ sector in order to make the symmetry as the symmetry of the Lagrangian. However, phase transformation for the left-handed neutrino fields are still allowed:

$$\Psi_L = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \rightarrow \begin{pmatrix} e^{-i\theta_1} & 0 & 0 \\ 0 & e^{-i\theta_2} & 0 \\ 0 & 0 & e^{-i\theta_3} \end{pmatrix} \Psi_L \rightarrow U_L \Psi_L, \quad (9)$$

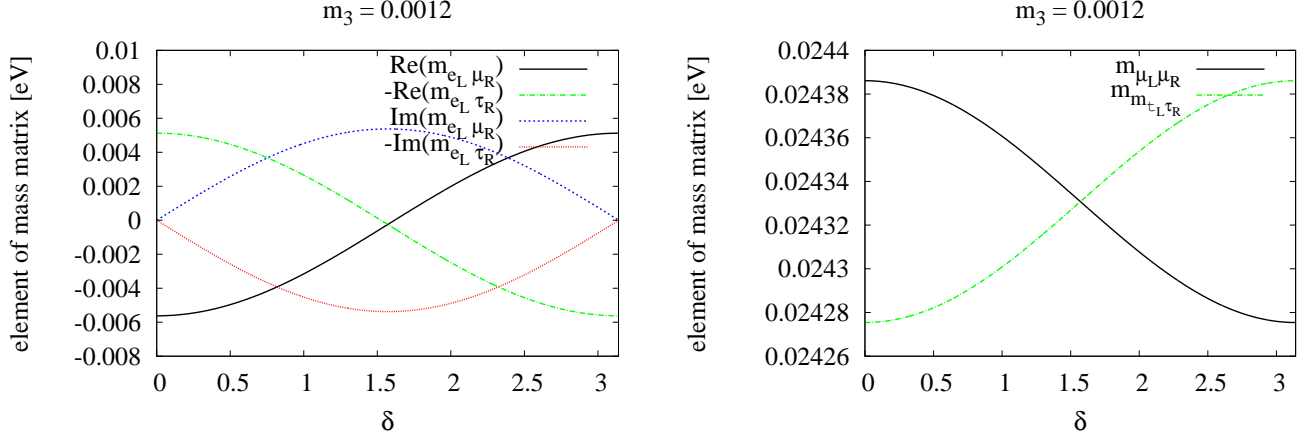


Figure 2: Left panel: The real and imaginary part of the elements $m_{e_L \mu_R}$ and $m_{e_L \tau_R}$ of the Dirac neutrino mass matrix in Eq. 3 as a function of δ (in radian) for $m_3 = -1.198 \times 10^{-3}$. Right panel: The diagonal elements $m_{\mu_L \mu_R}$ and $m_{\tau_L \tau_R}$ of the Dirac neutrino mass matrix in Eq. 3 as a function of δ for $m_3 = -1.198 \times 10^{-3}$. We have used IH central values for the Δm_{21}^2 , Δm^2 , s_{12}^2 and s_{13}^2 from Table 1 and for s_{23}^2 , we choose $s_{23}^2 = 0.5$.

We have demanded the invariance under simultaneous transformations $\Psi_R \rightarrow U_R \Psi_R$ and $\Psi_L \rightarrow U_L \Psi_L$ followed by a complex conjugation of the couplings. Complex conjugation of the couplings is equivalent to making a CP transformation. In the rest of this article, the symmetry under above mentioned transformations followed by a CP transformation is denoted as $\nu_{\mu R} - \nu_{\tau R}$ reflection symmetry. As a consequence of the $\nu_{\mu R} - \nu_{\tau R}$ reflection symmetry, we obtain the following matrix equation:

$$\left[U_L^\dagger \mathbf{M}_\nu^{\text{pheno}} U_R \right]^* = \mathbf{M}_\nu^{\text{pheno}}. \quad (10)$$

The most general solution of Eq. 10 is given by

$$\begin{aligned} \phi_1 &= n_1 \pi - \cos^{-1} [(-1)^{n_2} p] & \theta_1 &= n_1 \pi + \cos^{-1} [(-1)^{n_2} p]; \\ \phi_2 &= n_2 \pi & \theta_2 &= \cos^{-1} [(-1)^{n_2} p]; \\ \phi_2 &= \left(n_3 + \frac{1}{2} \right) \pi & \theta_2 &= \sin^{-1} [(-1)^{n_3} q] \end{aligned} \quad (11)$$

and

$$\eta = \frac{n\pi}{2}; \quad (12)$$

where, n , n_1 , n_2 and n_3 are arbitrary integers. The trivial solution ($n_1 = 0$, $n_2 = 0$ and $n_3 = 0$) of Eq. 10 physically corresponds to a symmetry under interchange of $\nu_{\mu R} \leftrightarrow -i\nu_{\tau R}$ followed by a CP transformation with $\eta = 0^0, 90^0, 180^0, 270^0, \dots$. However, the phenomenological neutrino mass matrix under consideration (Eq. 6 and Eq. 7) corresponds to $\eta = 272.6^0$. Therefore, tiny violation of the symmetry under interchange of $\nu_{\mu R} \leftrightarrow -i\nu_{\tau R}$ followed by a CP transformation is required to satisfy all the experimental results.

To summarize, we have considered Dirac neutrino mass matrix and investigated the possible symmetries in the $\nu_{\mu R}$ - $\nu_{\tau R}$ sector. In order to ensure that the imposed condition is a symmetry of the Lagrangian (not only the symmetry of the neutrino mass matrix in the flavor basis), we have restricted the requirements only to the singlet right-handed muon and tau neutrinos. Assuming the hermiticity of the neutrino mass matrix, we have obtained a particular structure of the phenomenological Dirac neutrino mass matrix with only 4 parameters. This 4 parameter Dirac neutrino mass matrix can explain all five (two squared-mass differences and three mixing angles) experimental results in the neutrino sector with particular predictions for the absolute values of the neutrino masses ($m_1 = 4.81 \times 10^{-2}$, $m_2 = 4.89 \times 10^{-2}$ and $m_3 = -1.198 \times 10^{-3}$ eV) and CP violating phase $\delta = 270^\circ$. We have shown that the 4 parameters phenomenological mass matrix corresponds to a symmetry under interchange of $\nu_{\mu R} \leftrightarrow -i\nu_{\tau R}$ followed by a CP transformation with a tiny violation of this symmetry to accomodate a value of the phase $\delta = 272.6^\circ$ as required by the mass matrix in Eq. (6).

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